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Comparison of SO(10)-Symmetric Fermion Mass Matrices with and without Degenerate Neutrinos

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Abstract

It has been recently suggested by others that one can simultaneously explain the depletions of solar electron-neutrinos and atmospheric muon-neutrinos along with a 7 eV neutrino component of mixed dark matter by postulating the existence of nearly-degenerate 2 eV neutrinos with the correct mixing parameters. We study this claim in the framework of a simple SO(10)-symmetric model constructed from the low scale data using a bottom-up procedure recently advanced by the authors and compare the results with and without degenerate neutrinos.

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In the past several years, three sets of observations, two direct and one indirect, suggest that neutrinos have mass with their masses and mixing parameters lying in restricted regions of the mixing planes according to certain user-preferred interpretations.

1) The depletion of solar neutrinos has now been observed[1] in experiments sensitive to the p-p, ^7Be and ^8B electron-neutrinos with the preferred[2] particle physics interpretation that these neutrinos undergo resonant conversion into muon-neutrinos in passing through the dense solar matter. This non-adiabatic Mikheyev-Smirnov-Wolfenstein (MSW) effect[3] is restricted to the central region of the 12 mixing plane where

$$\delta m^2_{12} = 5 \times 10^{-6}\ \mathrm{eV}^2, \qquad \sin^2 2\theta_{12} = 8 \times 10^{-3} \tag{1a}$$

2) The apparent depletion of the atmospheric muon-neutrino flux relative to the atmospheric electron-neutrino flux has now been observed[4] by three experimental collaborations and is widely interpreted as due to the oscillation of muon-neutrinos into tau-neutrinos during their passage through the atmosphere.[2] Although this phenomenon is considered to be on more shaky ground than the solar depletion effect, several recent refined flux calculations[5] have further restricted the mixing region but have not ruled out this interpretation. A typical point in the 23 mixing plane is represented by

$$\delta m^2_{23} = 2 \times 10^{-2}\ \mathrm{eV}^2, \qquad \sin^2 2\theta_{23} = 0.5 \tag{1b}$$

3) The indirect evidence for neutrino mass is strengthened somewhat by the recent COBE observation[6] of density fluctuations in the universe, for which the cocktail model[7] attributes the 30% hot dark matter component to neutrinos provided

$$\sum_i m_{\nu,i} \simeq 7\ \mathrm{eV} \tag{1c}$$

If the tau-neutrino is naturally assumed to be the heaviest with a mass of 7 eV, the present accelerator data[8] limit the mixing in the 23 plane to the region

$$m_{\nu_\tau} \simeq 7 \text{ eV}, \qquad \sin^2 2\theta_{23} \lesssim 4 \times 10^{-3} \tag{1d}$$

It is clear from the above that one can not explain all three effects with the interpretations cited in the framework of three non-degenerate, light left-handed neutrinos. One possibility is to introduce a new light sterile neutrino,[9] but a more economical recent suggestion[10] makes the assumption the three light neutrinos are nearly degenerate with masses close to 2 eV. In this way, the three neutrinos mix to give the solar and atmospheric depletions with the correct δm^2_{ij}'s, while they share equally in providing the hot component of mixed dark matter.

In this short note, we extend our recent construction[11] of SO(10)-symmetric mass matrix models for quarks and leptons to the case of degenerate neutrinos and compare our results with the non-degenerate cases considered earlier. Our bottom-up procedure allows us to start from the complete set of quark and lepton masses and two mixing matrices defined at the low scales and to reconstruct numerically quark and lepton mass matrices at the grand unification scale which yield the low scale results. By choosing the bases judiciously, we can single out mass matrices which exhibit simple SO(10) structure with as many texture zeros as possible from which simple mass matrix models can be identified. It is interesting to note that our findings in the non-degenerate case pointed to a greater simplicity for the mass matrix model incorporating observations 1) and 2) above rather than 1) and 3).

We shall summarize briefly the input and procedure and refer the interested reader to Ref. [11] for more of the details. From the information given in (1a) and (1b), we have taken for the lepton input in the non-degenerate (ND) case

$$\begin{aligned} m^{ND}_{\nu_e} &= 0.5 \times 10^{-6}\ \text{eV}, & m_e &= 0.511\ \text{MeV} \\ m^{ND}_{\nu_\mu} &= 0.224 \times 10^{-2}\ \text{eV}, & m_\mu &= 105.3\ \text{MeV} \\ m^{ND}_{\nu_\tau} &= 0.141\ \text{eV}, & m_\tau &= 1.777\ \text{GeV} \end{aligned} \tag{2a}$$

and

$$V_{LEPT} = \begin{pmatrix} 0.9990 & 0.0447 & (-0.690 - 0.310i) \times 10^{-2} \\ -0.0381 - 0.0010i & 0.9233 & 0.3821 \\ 0.0223 - 0.0030i & -0.3814 & 0.9241 \end{pmatrix} \tag{2b}$$

where we have assumed a value of for the electron-neutrino mass to which our analysis is not very sensitive and constructed the lepton mixing matrix by making use of the unitarity conditions. In the degenerate (DEG) case, we shall leave the mixing matrix unchanged and simply replace the set of light neutrino masses above by the new set

$$\begin{aligned} m^{DEG}_{\nu_e} &= (2.0 - 0.125 \times 10^{-5})\ \text{eV} \\ m^{DEG}_{\nu_\mu} &= 2.0\ \text{eV} \\ m^{DEG}_{\nu_\tau} &= (2.0 + 0.500 \times 10^{-2})\ \text{eV} \end{aligned} \tag{2c}$$

which yield the correct δm^2_{ij}'s with $\sum_i m_{\nu,i} \simeq 6$ eV.

For the quark input data, we evaluated the light quark masses at 1 GeV and the heavy quark masses at their running masses according to

$$\begin{aligned} m_u(1\text{GeV}) &= 5.1\ \text{MeV}, & m_d(1\text{GeV}) &= 8.9\ \text{MeV} \\ m_c(m_c) &= 1.27\ \text{GeV}, & m_s(1\text{GeV}) &= 175\ \text{MeV} \\ m_t(m_t) &= 150\ \text{GeV}, & m_b(m_b) &\simeq 4.25\ \text{GeV} \end{aligned} \tag{3a}$$

corresponding to a pole mass of $m_t^{phys} \sim 160$ GeV for the top quark. We adopted the following central values for the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix[12] at the weak scale

$$V_{CKM} = \begin{pmatrix} 0.9753 & 0.2210 & (-0.283 - 0.126i) \times 10^{-2} \\ -0.2206 & 0.9744 & 0.0430 \\ 0.0112 - 0.0012i & -0.0412 - 0.0003i & 0.9991 \end{pmatrix} \qquad (3b)$$

where we assumed a value of 0.043 for V_{cb} and applied strict unitarity to determine V_{ub}, V_{td} and V_{ts}.

The masses and mixing matrices were then evolved to the GUT scale by means of the one-loop renormalization group equations[13] for the minimal supersymmetric standard model (MSSM). By means of Kusenko's method[14] extended to leptons as well as quarks, one can then construct complex symmetric quark, charged lepton and light neutrino mass matrices which follow from the information at the GUT scale. The variation of two parameters controlling the choice of quark and lepton bases enabled us to identify mass matrices which exhibit simple SO(10) symmetry with a maximum number of texture zeros. The up, down, charged lepton and Dirac neutrino mass matrices in the non-degenerate case received contributions from two **10**'s and two **126**'s of SO(10) according to

$$\begin{aligned} M^U &\sim M^{N_{Dirac}} \sim diag(126;\ 126;\ 10) \\ M^D &\sim M^E \sim \begin{pmatrix} 10', 126 & 10', 126' & 10' \\ 10', 126' & 126 & 10' \\ 10' & 10' & 10 \end{pmatrix} \end{aligned} \qquad (4)$$

From the numerical results and these simple forms, the following matrix model emerged

$$\begin{aligned} M^U &= diag(F',\ E',\ C') & M^{N_{Dirac}} &= diag(-3F',\ -3E',\ C') \\ M^D &= \begin{pmatrix} 0 & A & D \\ A & E & B \\ D & B & C \end{pmatrix} & M^E &= \begin{pmatrix} F & 0 & D \\ 0 & -3E & B \\ D & B & C \end{pmatrix} \end{aligned} \qquad (5)$$

where only D is complex and the constraint, $4F'/F = -E'/E$, holds. There are nine independent parameters present along with four texture zeros for both the quark and lepton mass matrices.

If we now repeat the calculation for the degenerate case, replacing the input in (2a) by that in (2c), we find exactly the same numerical results for the up, down and charged lepton mass matrices with the same choice of bases; only the light or effective neutrino mass matrix is altered. The results for the non-degenerate and degenerate cases are, respectively,

$$M_{ND}^{N_{eff}} = \begin{pmatrix} (.4839 + .1534i) \times 10^{-4} & (-.9059 - .1304i) \times 10^{-3} & (.3023 + .0374i) \times 10^{-2} \\ (-.9059 - .1304i) \times 10^{-3} & (.1465 - .0001i) \times 10^{-1} & (-.5065 + .0002i) \times 10^{-1} \\ (.3023 + .0374i) \times 10^{-2} & (-.5065 + .0002i) \times 10^{-1} & 0.1502 \end{pmatrix} \tag{6a}$$

and

$$M_{DEG}^{N_{eff}} = \begin{pmatrix} 2.322 & (-.4714 + .0152i) \times 10^{-2} & (.0180 + .1202i) \times 10^{-1} \\ (-.4714 + .0152i) \times 10^{-2} & 2.370 & (-.1832 + .0225i) \times 10^{-2} \\ (.0180 + .1202i) \times 10^{-1} & (-.1832 + .0225i) \times 10^{-2} & 2.3752 \end{pmatrix} \tag{6b}$$

in units of electron volts.

A striking difference is observed between the two matrices. While a hierarchy similar to that for the down quark or charged lepton mass matrix is found in the non-degenerate case, when the neutrinos are nearly degenerate a larger diagonal matrix proportional to the identity is superposed on a smaller and more uniform background matrix. The conventional interpretation in the non-degenerate case is the operation of a seesaw mechanism,[15] whereby an extremely large right-handed Majorana neutrino mass matrix M^R gives rise to the observed light neutrinos according to

$$M_{ND}^{N_{eff}} = -M^{N_{Dirac}}(M^R)^{-1}M^{N_{Dirac}T} \tag{7a}$$

The suggestion by a number of authors[10] in the degenerate case is that, an additional diagonal and family-independent contribution arises from a left-handed Majorana neutrino mass matrix according to

$$M_{DEG}^{N_{eff}} = M^L - M^{N_{Dirac}}(M^R)^{-1}M^{N_{Dirac}T} \tag{7b}$$

Although such a term has been disfavored since it requires a Higgs triplet at tree level, the new suggestion is that M^L arises through a nonrenormalizable dimension-five operator of the form

$$M^L = m_L I = \lambda_L \nu_L^T \frac{<\phi>^2}{M} \nu_L \tag{8}$$

which yields the right order of magnitude with M one of the heavy right-handed Majorana masses and $<\phi>$ the Higgs expectation value at the weak symmetry-breaking scale.

If we now apply (7a) or (7b) making use of (6a) or (6b), respectively, and the diagonal Dirac neutrino matrix suggested by the model matrices in (4); we can determine the required right-handed Majorana matrix M^R for each scenario. From Ref. [11] where the best values for C', E' and F' are given, we find numerically

$$M_{ND}^R = \begin{pmatrix} (.1744 - .0044i)\times 10^{10} & (-.2332 + .0153i)\times 10^{11} & (-.2811 - .1925i)\times 10^{12} \\ (-.2332 + .0153i)\times 10^{11} & (.6773 - .0329i)\times 10^{12} & (-.1189 + .0243i)\times 10^{14} \\ (-.2811 - .1925i)\times 10^{12} & (-.1189 + .0243i)\times 10^{14} & (.6045 + .0624i)\times 10^{15} \end{pmatrix} \tag{9a}$$

for the non-degenerate case and

$$M_{DEG}^R = \begin{pmatrix} (-.4320 - .1444i)\times 10^{7} & (.1227 + .0253i)\times 10^{9} & (-.0730 + .3750i)\times 10^{11} \\ (.1227 + .0253i)\times 10^{9} & (-.2185 - .0032i)\times 10^{11} & (.0158 - .1057i)\times 10^{13} \\ (-.0730 + .3750i)\times 10^{11} & (.0158 - .1057i)\times 10^{13} & (.4634 + .1903i)\times 10^{14} \end{pmatrix} \tag{9b}$$

for the degenerate case in units of GeV. We have used a value of $m_L = 2.322$ eV at the GUT scale in order to minimize the hierarchy in the eigenvalues of M_{DEG}^R. The eigenvalues of the above matrices for the two cases are then found to be

$$
\begin{aligned}
M^{R_1}_{ND} &= 0.213 \times 10^{9} \text{ GeV} & \qquad M^{R_1}_{DEG} &= 0.165 \times 10^{9} \text{ GeV} \\
M^{R_2}_{ND} &= 0.475 \times 10^{12} \text{ GeV} & \qquad M^{R_2}_{DEG} &= 0.277 \times 10^{10} \text{ GeV} \\
M^{R_3}_{ND} &= 0.608 \times 10^{15} \text{ GeV} & \qquad M^{R_3}_{DEG} &= 0.501 \times 10^{14} \text{ GeV}
\end{aligned}
\tag{10a}
$$

If one uses (4), (6b) and (7a) without the diagonal M_L contribution, a much larger hierarchy appears in the degenerate case given by

$$
\begin{aligned}
M^{R_1} &= 0.469 \times 10^{4} \text{ GeV} \\
M^{R_2} &= 0.469 \times 10^{9} \text{ GeV} \\
M^{R_3} &= 0.490 \times 10^{13} \text{ GeV}
\end{aligned}
\tag{10b}
$$

Another interesting observation pertains to the structures of the matrices in (9a) and (9b). The matrix for the non-degenerate scenario was observed in Ref. [11] to have a near geometric texture,[16] i.e., a superposition of two geometric forms (with some elements zero), which suggests two **126** contributions, as approximated by

$$
M^R_{ND} = \begin{pmatrix} F'' & -\frac{2}{3}\sqrt{F''E''} & -\frac{1}{3}\sqrt{F''C''}e^{i\phi_{D''}} \\ -\frac{2}{3}\sqrt{F''E''} & E'' & -\frac{2}{3}\sqrt{E''C''}e^{i\phi_{B''}} \\ -\frac{1}{3}\sqrt{F''C''}e^{i\phi_{D''}} & -\frac{2}{3}\sqrt{E''C''}e^{i\phi_{B''}} & C'' \end{pmatrix} \tag{11a}
$$

where $E'' = \frac{2}{3}\sqrt{F''C''}$ and $\phi_{B''} = -\phi_{D''}/3$. For the degenerate case with $m_L = 2.322$ eV on the other hand, we find after a rephasing of the matrix

$$
M^R_{DEG} = \begin{pmatrix} F'' & -0.4\sqrt{F''E''} & -2.5\sqrt{F''C''}e^{i\phi_{D''}} \\ -0.4\sqrt{F''E''} & E'' & \sqrt{E''C''}e^{i\phi_{B''}} \\ -2.5\sqrt{F''C''}e^{i\phi_{D''}} & \sqrt{E''C''}e^{i\phi_{B''}} & -C'' \end{pmatrix} \tag{11b}
$$

where $E'' = 1.4\sqrt{F''C''}$, with $\phi_{D''}$ and $\phi_{B''}$ near $\pm 90^{\circ}$. However, the simple superposition structure of two geometric forms is lost. This situation can only be improved somewhat by varying m_L at the expense of introducing a much larger hierarchy in the eigenvalues for M^R_{DEG}. In the non-degenerate case, only three additional parameters need be added to the nine already counted in the model, while in the degenerate case, seven parameters (m_L, four magnitudes and two phases) must be added.

Several other authors have recently analysed[17] the degenerate case, but they employed real diagonal matrices for the up quark, Dirac neutrino and right-handed Majorana neutrino mass matrices resulting in no simple SO(10) Higgs structure at the GUT scale. For their choice of matrices, eighteen parameters are required if the proper phases are included. In this note we have demonstrated that a mass matrix model can be constructed with fewer parameters, not only for the non-degenerate neutrino scenario but for the degenerate one as well, which exhibits a simple SO(10) structure with four texture zeros in the quark and in the lepton mass matrices. The simplicity arises in the basis where the up quark, and hence Dirac neutrino, mass matrices are diagonal, but the light neutrino and right-handed Majorana matrices are non-diagonal and complex-symmetric.[18]

The basic change in going from the non-degenerate to degenerate case arises in the light neutrino mass matrix at the GUT scale, which suggests the presence of a family-independent, family-diagonal left-handed Majorana matrix and a modified heavy right-handed Majorana mass matrix. But the observed superposition of two **126** geometric matrices for the right-handed Majorana matrix is lost with the introduction of at least four more parameters required than in the non-degenerate case.

These complicating features suggest to us that the non-degenerate scenario where the solar and atmospheric neutrino depletions are readily understood in the simplest SO(10) framework with the fewest number of parameters remains the most viable theoretical one at present, provided the COBE results[6] can be interpreted entirely in terms of cold dark matter. Future experiments which can further explore the limits on double beta decay[19] and can heroically lower the present upper limit of 7.3 eV on the electron-neutrino mass[20] by one order of magnitude will definitively clarify this issue.

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References

[1] R. Davis et al., Phys. Rev. Lett. **20**, 1205 (1968); in *Neutrino '88*, ed. J. Schnepp et al. (World Scientific, 1988); K. Hirata et al., Phys. Rev. Lett. **65**, 1297, 1301 (1990); A. I. Abazov et al., Phys. Rev. Lett. **67**, 3332 (1991); Phys. Rev. C **45**, 2450 (1992); P. Anselmann et al., Phys. Lett. B **285**, 376, 390 (1992).

[2] G. L. Fogli, E. Losi and D. Montanino, CERN preprint, CERN-TH-6944-93; S. T. Petcov, SISSA preprint, SISSA-2-94 EP; N. Hata and P. Langacker, University of Pennsylvania report, UPR-0592-T.

[3] S. P. Mikheyev and A. Yu Smirnov, Yad Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1986)]; Zh. Eksp. Teor. Fiz. **91**, 7 (1986) [Sov. Phys. JETP **64**, 4 (1986)]; Nuovo Cimento **9C**, 17 (1986); L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); **20**, 2634 (1979).

[4] K. S. Hirata et al., Phys. Lett. B **205**,416 (1988); **280**, 146 (1992); and **283**, 446 (1992); D. Casper et al., Phys. Rev. Lett. **66**, 2561 (1991); and Phys. Rev. D **46**, 3720 (1992); W. W. M. Allison et al., Argonne preprint ANL-HEP-CP-93-32.

[5] Two recent studies of the atmospheric neutrino flux issues are reported in T. K. Gaisser, Phys. Rev. D **47**, 4823 (1993); D. H. Perkins, Oxford preprint OUNP-93-32.

[6] G. F. Smoot et al., Astrophys. J. **396**, L1 (1992).

[7] Q. Shafi and F. Stecker, Phys. Rev. Lett. **53**, 1292 (1984); Ap. J. **347**, 575 (1989); E. L. Wright et al., Astrophys. J. **396**, L13 (1992); M. Davis, F. J. Summers and D. Schagel, Nature **359**, 393 (1992); A. N. Taylor and M. Rowan-Robinson, ibid. **359**, 396 (1992); R. K. Shaefer and Q. Shafi, ibid. **359**, 199 (1992).

[8] N. Ushida et al., Phys. Rev. Lett. **57**, 2897 (1986).

[9] J. T. Peltoniemi, D. Tommasini and J. W. F. Valle, Phys. Lett. B **298**, 383 (1993); J. T. Peltoniemi and J. W. F. Valle, Nucl. Phys. B **406**, 409 (1993); E. Akhmedov, Z. Berezhiani, G. Senjanovic and Z. Tao, Phys. Rev. D **47**, 3245 (1993).

[10] D. Caldwell and R. Mohapatra, Phys. Rev. D **48**, 3259 (1993); S. T. Petcov and A. Yu. Smirnov, Phys. Lett. B **322**, 109 (1994); A. S. Joshipura,Ahmedabad preprint, PRL-TH/93/20.

[11] C. H. Albright and S. Nandi, Fermilab-PUB-93/316-T and Fermilab-PUB-94/061-T and OSU Preprints 282 and 286, submitted for publication.

[12] Review of Particle Properties, Phys. Rev. D **45**, No. 11, Part II, June (1992); cf. also S. Stone, to appear in the Proceedings of the Yukawa Workshop held at the University of Florida, Feb. 11-13, 1994.

[13] S. Naculich, Phys. Rev. D **48**, 5293 (1993).

[14] A. Kusenko, Phys. Lett. B **284**, 390 (1992).

[15] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supersymmetry*, edited by P. Van Nieuwenhuizen and D. Z. Freedman (North-Holland, Amsterdam, 1979); T. Yanagida, Prog. Theor. Phys. **B 315**, 66 (1978).

[16] E. H. Lemke, Mod. Phys. Lett. A **7**, 1175 (1992).

[17] P. Bamert and C. P. Burgess, McGill preprint, McGill-94/07; D.-G. Lee and R. N. Mohapatra, U. of Maryland preprint, UMD-PP-94-95; A. Ioannissyan and J. W. F. Valle, Valencia preprint, IFIC/94-05; A. S. Joshipura, Ahmedabad preprint, PRL-TH-94/08.

[18] A general analysis of the number of phases required in the various quark and lepton mass matrices has been carried out in A. Kusenko and R. Shrock, Stony Brook preprints ITP-SB-93-58,62,63; cf. also A. Santamaria, Phys. Lett. B **305**, 90 (1993).

[19] M. Beck et al., Phys. Rev. Lett. **70**, 2853 (1993).

[20] R. G. H. Robertson et al., Phys. Rev. Lett. **67**, 957 (1991).